NAME: ______________________________

Review of Integration
Math 16020

1 Antiderivatives

(a) $\int x^n\,dx =$ ______________________ (where $n \neq$ ____)

(b) $\int \frac{1}{x}\,dx =$ ______________________

(c) $\int a\,dx =$ ______________________ (where a is a constant)

(d) $\int \sin(x)\,dx =$ ______________________

(e) $\int \cos(x)\,dx =$ ______________________

(f) $\int \sec^2(x)\,dx =$ ______________________

(g) $\int \sec(x)\tan(x)\,dx =$ ______________________

(h) $\int e^x\,dx =$ ______________________

(i) $\int af(x)\,dx =$ ______________________ (where a is a constant)

(j) $\int \left(f(x) + g(x)\right)\,dx =$ ______________________

(k) $\int \left(f(x) - g(x)\right)\,dx =$ ______________________

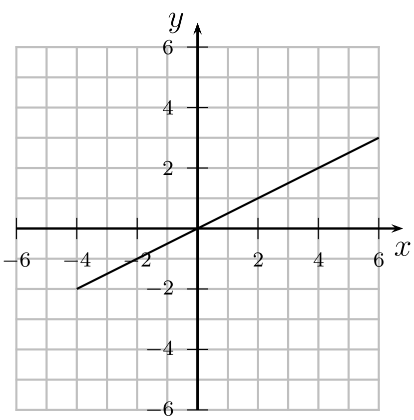
Example 1. $\int \left(2e^x + \frac{5}{x} + \cot(x)(\csc(x) + \sin(x))\right)\,dx$

2 Definite Integrals

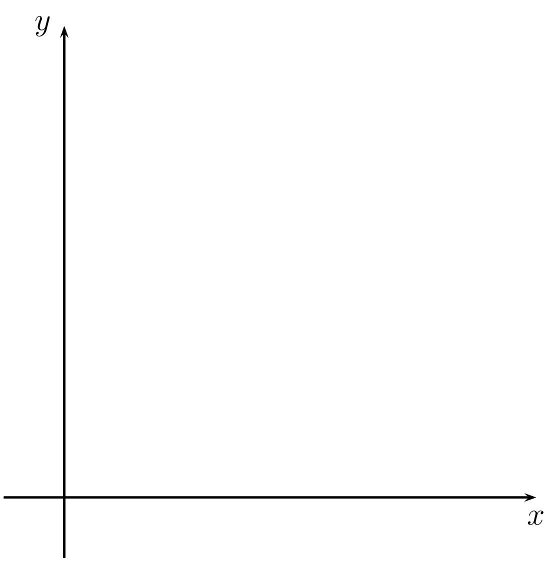
- Antideriavtives (no limits on $\int$) are ________________________.
- Definite integrals (number limits on $\int$) are ________________________.

The **geometric meaning** of $\int_a^b f(x)\,dx$ is the ________________________ between $y = f(x)$ and the ________________________, from $x =$ ____________ to $x =$ ____________.

Example 2. Find $\int_{-4}^{6} \frac{1}{2}x\,dx$ geometrically.



Example 3. Set up a definite integral that represents the area of the region bounded by $y = 3x+1$, $y = 0$, $x = 2$, and $x = 7$



3 The Fundamental Theorem of Calculus

Theorem 4 (Fundamental Theorem of Calculus, Part II)**.** *If f is continuous on the interval $[a, b]$ and F is any antiderivative of f, then*

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Theorem 5 (Net Change Theorem)**.** *If F is differentiable on the interval $[a, b]$, then*

$$\int_a^b F'(x)\,dx = F(b) - F(a)$$

Example 6. Evaluate the definite integrals.

(a) $\displaystyle\int_1^4 x^2\left(x + \frac{1}{\sqrt{x}}\right)\,dx$

(b) $\displaystyle\int_0^{16} \frac{\sqrt[4]{x^3}}{5}\,dx$

Example 7. At noon, a hot air balloon pilot begins to fill his balloon with air at a rate of

$$r(t) = 1000t^2$$

where t is measured in hours since noon and $r(t)$ is measured in cubic feet per hour.

(a) How much air goes into the balloon between 1:00PM and 2:00PM?

(b) Approximately how many hours does it take to fill the hot air balloon? (Hot air balloons typically hold $77,000$ cubic feet of air.) Round your answer to the nearest hundredth of an hour.

Example 8. If $y' = \sin(x)$ and $y(0) = 5$, find $y\left(\frac{\pi}{6}\right)$.